Validation of the virtual experiment of the tilted-wave interferometer

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The tilted-wave interferometer (TWI) is a promising device for measuring the form of aspheres. However, its complexity necessitates the embedding of a virtual experiment (VE). At the Physikalisch-Technische Bundesanstalt (PTB), the VE is validated by measuring a calibrated reference surface, i.e. a spherical section with traceable radius and sphericity measurements.

1 Introduction

The TWI combines a special setup (see (Fig. 1)) with model-based data evaluation [1]. The setup contains a microlens array (MLA) with a blocking mask in the object arm together with a beam stop in the Fourier plane of the imaging optics. To reconstruct the form from these complex data, a computational model of the whole setup is used to simulate the data for the design of the surface under test (SUT). Therefore, VEs play a crucial role in this application and the validation of the VEs is an important step for the development of a traceable reference system for the form measurement of aspheres. Within the European project ViDiT (Trustworth Virtual Experiments and Digital Twins), the VE of the TWI is validated by measurement of a reference surface with calibrated radius and sphericity, i.e. its deviation from an ideal spherical surface, and comparison to these reference results [2]. In this contribution, the validation method is demonstrated, and the validation results are presented. For further details about the measurement system, we refer to [1].

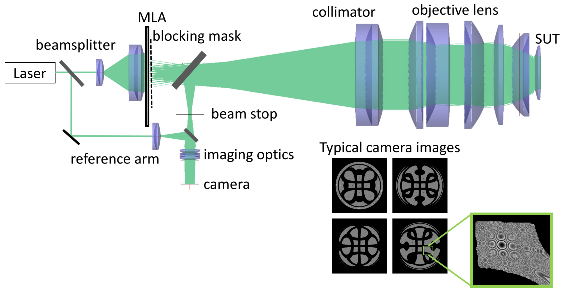

Fig. 1 *Sketch of the TWI setup together with typical camera images for an asphere (adapted from [1], CC BY 4.0).*

2 Reference Measurements

A spherical reference surface with a design radius of 30 mm and a diameter of 50 mm is used. The radius and the sphericity are calibrated in two different setups of PTB. In order to compare these two surface characteristics to the absolute form measured by the TWI, several processing steps are necessary: the absolute form of the SUT has to be calculated from the radius and sphericity on the diameter measured by the reference instruments. The resulting absolute form is then interpolated to a common 1000 x 1000 pixel grid of the evaluation diameter chosen for the TWI measurement. Then, the best-fit sphere is calculated based on the interpolated data for the common evaluation diameter and subtracted from the original data. The reference radius used for comparison is then taken from the best-fit sphere and the sphericity is interpolated to the common regular grid (same grid as above). To account for this processing in the uncertainty budget of the reference measurements, these steps are applied to $N = 20000$ samples generated for the deviations to the sphericity and radius according to the original uncertainty budget of the reference measurements.

3 TWI Measurements

For the TWI measurements, an analogous processing is required to ensure comparability with the reference surface characteristics. First, the absolute form of the SUT is interpolated to the same regular grid as used for the reference measurement. Then, the best-fit sphere based on the interpolated data is calculated. The resulting radius is the radius measured by the TWI. Afterwards, the best-fit sphere is removed from the original data points and the resulting sphericity is interpolated to the common regular grid. This is the sphericity measured by the TWI.

For the uncertainty evaluation, two methods developed within the project [3] have been applied. In general, the uncertainty evaluation for the TWI is challenged by the complexity of the nonlinear VE. As a reference method, an approximate Bayesian approach was developed [4]. For this method the uncertainty reflects the state of knowledge (Bayes), but the method has high computational complexity (approx. 16 h). As a second method, a law of propagation of uncertainty method (LPU) in accordance with the Guide to the Expression of Uncertainties

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(GUM) was developed. For this approach, the full VE is linearized. This method provides a fast evaluation (less than 10 min) while still being accurate due to the local linear behavior of the TWI. Both methods were applied for the TWI measurements of two different surface positions (confocal position (Conf., one patch due to nulltest of spherical surface) and second position (Sec., several patches)). It is important to note that only a limited number of input uncertainties with rough orders of magnitude were used to demonstrate the methods developed in the project.

4 Validation of Radius and Sphericity

The results of the validation of the sphericities are shown in Fig. 2. Deviations between the reference and TWI sphericity estimates are in the range of only a few ten nanometre. The total coverage is close to 95 %. The uncertainties of the reference measurement cover the estimates obtained by the TWI fairly well.

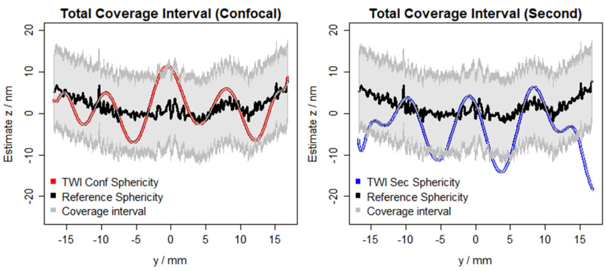

Fig. 2 *1D plots of the estimates taken at position x closest to 0. The plot on the left shows the estimates obtained using the confocal position and the plot on the right shows the estimates obtained using the second position within the TWI. Coverage refers to a normality assumption with coverage factor $k = 2$.*

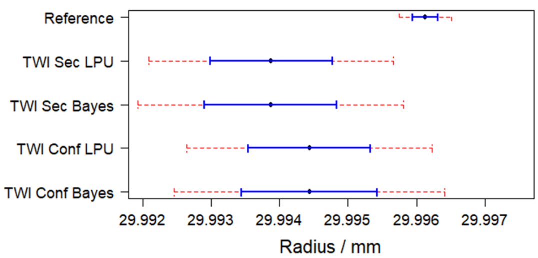

Fig. 3 *Error bar plot of the radius for the two TWI positions (Confocal - Conf and Second - Sec) with the two respective uncertainty evaluation methods (Bayes and LPU) and the reference instrument. The black dot indicates the estimate of the radius and the blue and red bars indicate the intervals referring to a normality assumption with coverage factor $k = 1$ and $k = 2$ respectively.*

The results of the validation of the radius are shown in Fig. 3. Overlap between the approaches of the TWI and the reference are already evident for most methods. Since the alignment of the SUT along the optical axis is a major source of uncertainty for the radius measured by the TWI, a procedure for alignment of the SUT along the optical axis of the TWI is being developed to reduce the influence. Furthermore, additional position errors along the optical axis, that are not yet considered in the uncertainty methods may lead to the systematic deviations visible in the results. Future adjustments will lead to a better agreement.

5 Discussion and Conclusion

The VE of the TWI can be validated using reference measurements of the radius and sphericity. In this contribution, the VE of the TWI and two different uncertainty methods were applied. While the uncertainties resulting from the TWI are rather low for the sphericity (not shown in Fig. 2) due to the currently limited number of influencing factors included in the methods, the estimates in conjunction with the obtained uncertainties are well covered by, and comparable to, the reference measurement and its uncertainty. Therefore, the VE of the TWI is considered validated for the computation of an estimate. For validation of the employed uncertainty methods, further research is required. In future work, influencing parameters not yet included must be added to the methods developed (e.g. uncertainties caused by model errors (e.g. due to form deviations of the reference surfaces during model correction), higher frequency reconstruction terms, ambient conditions). Furthermore, reliable determination of input uncertainties is subject to future research.

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