Uncertainty contribution of surface form errors in refractive index measurement of solid prisms

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The Physikalisch-Techniche Bundesanstalt (PTB) in Braunschweig, offers a calibration service for the refractive index n_{vac} of solid prisms. The measurement uncertainty of the refractive index measurement is directly linked to the uncertainty in the apex angle and the angle of minimum deviation. Therefore surface form errors on the prism's transmitting surfaces impact the measurement uncertainty. This contribution introduces a new modeling concept to estimate the refractive index uncertainty contribution of surface form errors, which are determined by an interferometer.

1 Introduction

A comprehensive investigation about sources of uncertainty in refractive index measurement employing the minimum deviation angle method was published in 1990 by Tentori and Lerma [1]. This article involved a simple but useful model for measurement errors in the prism's apex angle ϕ, by estimating a slope angle based on the prism length and its flatness error. The model in [1] does not provide a spatial resolution of apex angle errors e_ϕ, but is a reasonable estimate and explains, how the associated angle error e_{δ_m} of the minimum deviation angle may be determined and how it effects the refractive index. However, there are multiple calculation errors present in [1] concerning the provided example values for e_ϕ. In 2012 PTB published a proceeding [2] announcing its participation in a refractive index measurement comparison with other National Metrology Institutes (NMIs) (COOMET.PR-S3 [3]). This proceeding introduced the measurement principle of minium deviation angle refractometry at PTB and also the measurement uncertainty budget employed at the time. A follow up article [4] focused on the influence of surface form errors on refractive index measurements and methods for its modeling. The results presented in [4] are mostly consistent with the model proposed in [1] and therefore indicate, that the uncertainty contributions due to flatness errors reported in [2] may result in too small uncertainty values for certain samples. The results of the measurement comparison COOMET.PR-S3 [3] were published in 2016 showing significant inconsistencies between the measurements of the participating NMIs and thus strongly indicating an infeasible measurement uncertainty. Based on these observations the uncertainty budget for refractive index measurements was thoroughly revised in 2024 and now includes a spatially resolved evaluation of sample specific flatness errors causing an overall higher, but more reliable uncertainty estimation.

2 Spatially resolved uncertainty contributions

The equation to calculate the vacuum refractive index n_{vac} from the measured apex angle ϕ and measured minimum deviation angle δ_m is given in [1].

$$n_{\text{vac}} = n_{\text{air}} \cdot \frac{\sin\left([\phi + \delta_m]/2\right)}{\sin(\phi/2)} \qquad (1)$$

Form this equation follows the relation between the minimum deviation angle δ_m and the apex angle ϕ.

$$\delta_m = 2 \cdot \text{asin}\left(\frac{n_{\text{vac}}}{n_{\text{air}}} \cdot \sin(\phi/2)\right) - \phi \qquad (2)$$

Considering a spatially resolved apex angle error $e_{\phi_x}(x,y)$, the spatially resolved minimum deviation angle error is obtained by the derivative of Eq. (2).

$$e_{\delta_{m_x}}(x,y) = \frac{\partial \delta_m}{\partial \phi} \cdot e_{\phi_x}(x,y) \qquad (3)$$

The uncertainties u_{ϕ_x} and $u_{\delta_{m_x}}$ in dependence of the apex angle error $e_{\phi_x}(x,y)$ and minimum deviation angle error $e_{\delta_{m_x}}(x,y)$ are determined in dependence of the error's distribution functions as obtained from experimental data. Finally the resulting error $e_n(x,y)$ and corresponding uncertainty u_n in vacuum refractive index in dependence of the flatness error $W(x,y)$ is obtained by Eq. (4).

$$u_n = \sqrt{\left(\frac{\partial n_{\text{vac}}}{\partial \phi} u_{\phi_x}\right)^2 + \left(\frac{\partial n_{\text{vac}}}{\partial \delta_m} u_{\delta_m}\right)^2} \qquad (4)$$

To obtain $e_n(x,y)$ using Eq. (4), the uncertainties u_{ϕ_x} and $u_{\delta_{m_x}}$ are replaced with the errors $e_{\phi_x}(x,y)$ and $e_{\delta_{m_x}}(x,y)$. In a typical calibration measurement of solid prisms at PTB a triangular prism as depicted in Fig. 1 with side A and B enclosing the prism angle $\phi \in [45°, \dots, 65°]$, length $L_x > 50\,\text{mm}$ and height $H_y > 50\,\text{mm}$ and a vacuum refractive index $n_{\text{vac}} \in [1.5, \dots, 1.7]$ is measured. The flatness error on side A and B of the prism is measured with an interferometer. The light beam used in the refractometer covers a circular aperture of $D \approx 25\,\text{mm}$ centered on the prism surface when measuring the prism apex angle. The corresponding flatness error $W(x,y)$ of a prism is depicted in Fig. 2. The apex angle measurement is conducted by perpendicular alignment

DGaO Proceedings 2026 – http://www.dgao-proceedings.de – ISSN: 1614-8436 – urn:nbn:de:0287-2026-A004-8
https://doi.org/10.83229/127-a004 – submitted: 08.07.2026 – published: 07.09.2026

of the prism surface to an autocollimator [2, 4].

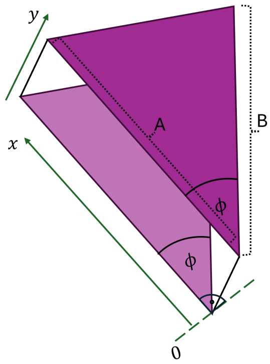

Fig. 1 *Schematic of a prism with side A and B enclosing the apex angle ϕ. The x and y axis are aligned perpendicular to the normal vector of the prism area (A or B) chosen for flatness measurement.*

Any flatness errors of the surface in x-direction will cause local apex angle errors $e_{\phi_x}(x,y)$. the prism angle error is therefore calculated by Eq. (5).

$$e_{\phi_x}(x,y) = \text{atan}\left(\frac{\partial W(x,y)}{\partial x}\right) \qquad (5)$$

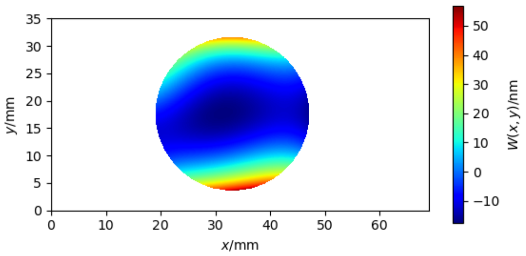

Fig. 2 *Flatness error $W(x,y)$ of one circular prism surface section.*

An example of the apex angle errors corresponding to the flatness error in Fig. 2 is depicted in Fig. 3. Using Eq. (3) and (4) the corresponding errors in the minimum deviation angle and the vacuum refractive index are calculated. Fig. 4 shows the local refractive index errors $e_n(x,y)$ determined based on the flatness errors $W(x,y)$ in Fig. 2.

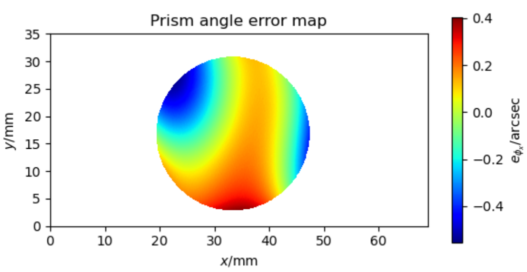

Fig. 3 *Prism apex angle error $e_n(x,y)$ of one circular prism surface section.*

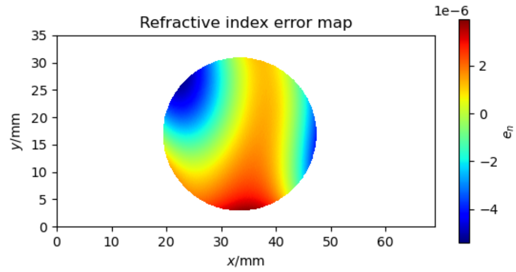

Fig. 4 *Refractive index error $e_n(x,y)$ of one circular prism surface section.*

The refractive index uncertainty for the individual circular section may be obtained from Eq. (4) if u_{ϕ_x} and $u_{\delta_{m_x}}$ are already identified, or directly from the distribution function of $e_n(x,y)$. For this purpose the histograms obtained from the spatially sampled $e_{\phi_x}(x,y)$, $e_{\delta_{m_x}}(x,y)$ and $e_n(x,y)$ are each approximated by a gaussian probability distribution function, such that at least 95% of the errors are covered by the gaussian curve. The standard deviation of the fitted normal distribution is used as an estimator of the respective uncertainty. For the example presented here in Fig. 2 - 4, a prism specified by $\phi \approx 62.05\,°$, length $L_x > 69\,\text{mm}$ and height $H_y > 35\,\text{mm}$ and a vacuum refractive index $n_{\text{vac}} \approx 1.73$ was measured. The resulting uncertainties in the circular section are $u_{\phi_x} \approx 0.21\,\text{arcsec}$, $u_{\delta_{m_x}} \approx 0.48\,\text{arcsec}$ and $u_n \approx 1.37 \cdot 10^{(-6)}$. The procedure needs to be repeated for side A and B of the prism to obtain the total uncertainty from flatness errors. Also, the high spatial resolution achieved with the interferogram can not be obtained by the refractometer. Therefore, the uncertainty calculation for the refractive index at PTB involves additional steps including a sweep of the region of interest on the prism surfaces and consideration of the mean value of the refractive index measured on individual circular apertures. A more detailed explanation will be published in a full length article.

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