

Signal Analysis for Phase Retrieval using the Wigner Distribution Function

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In this contribution we observe the application of the Wigner Distribution Function (WDF) for signal analysis after Phase Retrieval. The simultaneous representation of spatial and spectral coordinates allows for better identification and assessment of interference phenomena in the reconstructed image and diffraction planes.

1 Introduction

Phase Retrieval (PR) is a method for recovering the complex wavefront of a coherently illuminated object from its intensity distributions at various distances (defocus). In order to adapt the object size to the camera sensor we introduced an optical interface, which allows us to additionally control the aperture. The application of a tunable lens enables the shifting of the image plane along the optical axis, so that the different defocused distributions are captured at a fixed axial camera position. This stack of defocused images is iteratively processed by a PR Algorithm, so that from the complex wavefields the reconstruction in the image plane can be revealed. In order to estimate the result of the PR algorithm we compare the intensity distribution of reconstruction to the directly captured image intensity. There are high-frequency noise and artefacts visible.

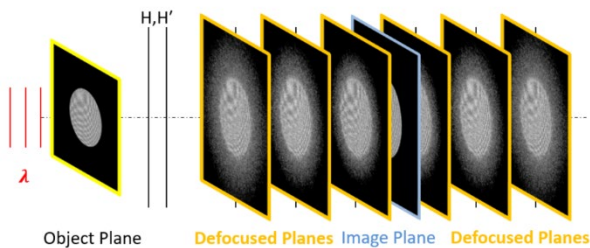


Fig. 1 Principle setup for Phase Retrieval.

2 Wigner Distribution Function

In this contribution we investigate the evaluation of the whole system performance (optical setup and PR Algorithm) by observing with the WDF. This function allows (i) the representation of spatial and spectral information simultaneously, (ii) the modeling of local/spatial variant transfer function and (iii) the derivation of Space-Bandwidth-Product.

Eq. (1) describes the development of WDF based on a spatial signal $u(x)$

$$W(x, \nu) = \int_{-\infty}^{+\infty} u\left(x + \frac{x'}{2}\right) u^*\left(x - \frac{x'}{2}\right) e^{-j2\pi\nu x'} dx' \quad (1)$$

and Eq. (2) based on a spectral signal $\tilde{u}(\nu)$:

$$W(x, \nu) = \int_{-\infty}^{+\infty} \tilde{u}\left(\nu + \frac{\nu'}{2}\right) \tilde{u}^*\left(\nu - \frac{\nu'}{2}\right) e^{+j2\pi\nu' x} d\nu' \quad (2)$$

with x as the spatial and ν as the frequency coordinate. The WDF is a representation of a 1-dimensional signal in 2 dimensions: one horizontal spatial axis x and one vertical spectral axis ν , respectively. Due to its convolution kernel within the integral and the following Fourier transform the number of samples N increases to $(2N - 1)^2$.

3 Signal analysis by means of WDF

The WDF exhibits its special properties after performing transforms or calculations. For instance, the numerical propagation (Fresnel transform) of a signal results in a horizontal shear of WDF.

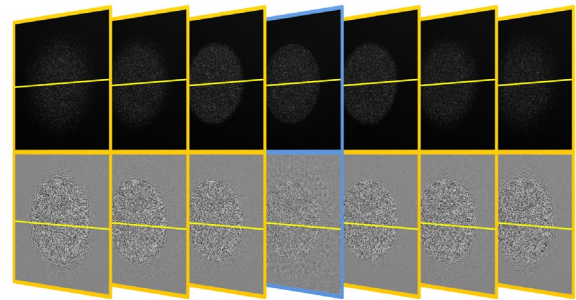


Fig. 2 Speckle distributions at defocused planes in intensity (top row) and phase (bottom row) after PR Algorithm. The WDF is determined along a linear cross-section (yellow line).

The WDF along a linear cross-section of a speckle distribution (Fig. 2) corresponds to a filled, speckled rectangle in image plane (Fig. 3). Defocusing leads to a distance-dependent horizontal shear. The red line marks the expected tangent of shear. Fig. 3 shows this shearing effect of reconstructed defocused distributions after the PR algorithm based on a simulated (row above) and on a measured speckle field (bottom row) whereas the WDF in the image plane (center) is not sheared. These speckled WDFs seem to be free from pattern to reveal some further information.

We consider now the behavior of structured specimen of a phase grating with a grating period of $50\mu\text{m}$ and a grating depth of 128nm at a wavelength of 633nm (Fig. 7). In Fig. 4 we examine the latest

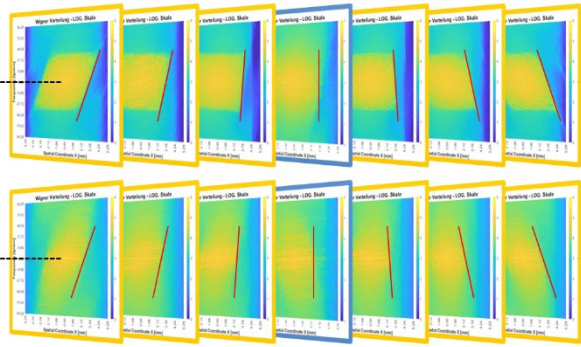


Fig. 3 WDF of linear cross-section of speckle field after PR Algorithm. Simulation (top row) vs. lab experiment (bottom row). Highlighted orange: defocused planes, highlighted blue: image plane.

defocused plane and compare its WDF (left) to the simulated one of the phase grating (right). The basic pattern of simulation can be found in the reconstruction, however, there are additional spectral components at lower frequencies (s. Fig. 4, red marked areas).

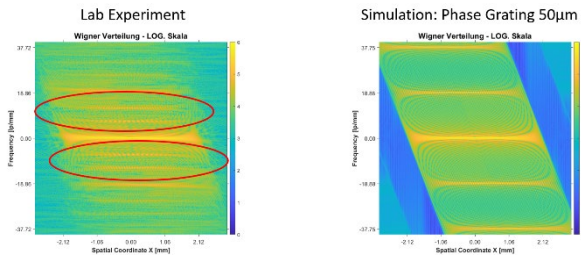


Fig. 4 Phase grating: WDF of experiment (left) vs. simulation (right). The red encircled areas contain unexpected spectral components.

A horizontal walk parallel to the spatial abscissa reveals pulses. A vertical further step away shows a multiple spectral component containing a train of narrower pulses and so on. This might be traced back to signal structure with a fundamental frequency and its harmonics, which is typical for any phase modulation. Indeed, we can recognize and confirm a beat pattern of nearly $238\mu\text{m}$ period in the reconstructed image plane as seen in Fig 7.

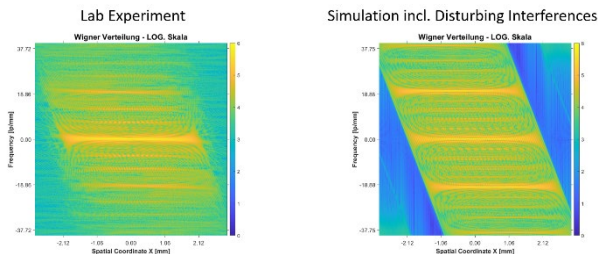


Fig. 5 Synthesis (right) of WDF from experiment (left).

In Fig. 5 we try to synthesize the experimental WDF by interfering this beat pattern and phase ramps corresponding to $\pm 1^{\text{st}}$ diffraction orders of the $50\mu\text{m}$ phase grating.

The importance of phase information is illustrated in Fig. 6. Here we compare the WDF of the

reconstruction (left) with and without phase (set to zero, right). If we omit the phase information, we cannot perform the previously mentioned analysis.

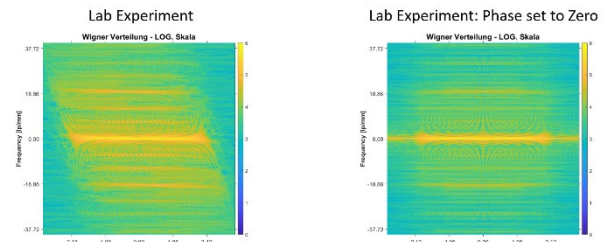


Fig. 6 WDF from experiment including phase vs. lost phase information (right).

It is important to note that we have just performed a signal evaluation. The origin of the disturbing structures needs to be clarified by a critical review of our PR algorithm (numerical issues, sampling, quantization) or setup (defocusing distances, interferences of reflections, multiple light propagation).

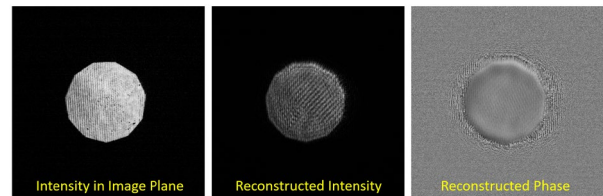


Fig. 7 Experiment: phase grating captured image (left), reconstructed intensity (center) and phase (right). Unexpected interferences occur in large areas of reconstruction.

4 Summary

We have shown that the WDF enables an extended signal analysis (beside the conventional separate spatial and spectral observation) and visualizes phase effects like propagation and phase modulation (harmonics, disturbing interferences) at the expense of a more elaborate presentation: A one-dimensional signal with N samples leads to a 2 dimensional WDF with $(2N - 1)^2$ samples. Due to the fact, that a finite observation window corresponds to an infinite distribution after Fourier forward/back transform the WDF always contains aliasing.

References

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