

Teaching quantum computing using a full adder as an example

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From a few experiments some basics of quantum physics and their mathematical background can be understood. We chose the full adder as an exemplary experiment to demonstrate how future engineers can learn about quantum computing and its applications.

1 Introduction

In 2017, the German national academies of science recommended integrating basic quantum education into engineering curricula [1]. Despite this directive, further development in the field has remained limited. To address this growing educational gap, we propose an initial approach for teaching quantum computing specifically to engineering students.

2 Interference Experiments: Classical and Quantum Mechanical

In a classical interference experiment [2] two monochromatic, electromagnetic waves with electric fields $\vec{E}_1$ and $\vec{E}_2$ superpose. The optical waves oscillate at such high frequency that detectors detect only the time averaged intensity (with $\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$ the total superimposed electric field):

$$\langle \vec{E}_{tot} \cdot \vec{E}_{tot} \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \vec{E}_{tot} \cdot \vec{E}_{tot} dt \quad (1)$$

Since the mean value is also a probabilistic value, this it is a first hint of probability.

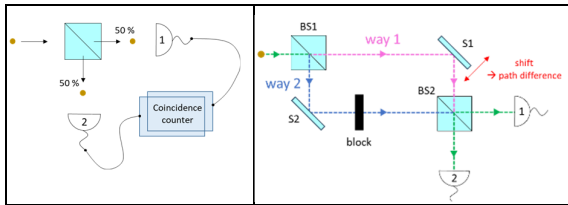


Fig. 1 Simplified GRA experiment. Left: Single photon hits a beam splitter. Right: expansion of the single photon experiment by an interferometer.

In 1982 Grangier, Roger und Aspect (GRA) [3] carried out single photon experiments – here shown in a very simple form (rather Hanbury Brown, Twiss experiment). As illustrated in Fig. 1, the setup consists of a 50:50 beamsplitter and two single-photon detectors. Upon entering the setup, the single photon is prepared in a superposition state of both paths. However, the subsequent measurement on the detectors collapses this state, ensuring the photon is detected at either detector 1 or detector 2, but never at both detectors simultaneously. This is in

contrast to classical wave theory that allows a simultaneous detection in both detectors with a certain probability.

A verified single-photon source can be utilized to perform fundamental interference experiments, such as a Michelson or Mach-Zehnder interferometer, see Fig. 1, right. Result: the detectors register single-photon interference as long as the photon's path remains indistinguishable. Once the 'which-way' information is accessible, the interference disappears.

3 Consequences and Mathematical Basics: Qubits

A quantum bit (qubit) is a two-state quantum mechanical state $|\Psi\rangle$ (see Fig. 1, right photon with way 1 or 2) and is the superposition of state $|0\rangle$ or state $|1\rangle$ ($\alpha, \beta \in \mathbb{C}$) [3]:

$$|\Psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle \quad (2)$$

The calculus will be easier, if both states are orthogonal to each other: $\langle 0|1\rangle = \langle 1|0\rangle = 0$. The same as in vector calculus and cartesian coordinates.

The total probability for measuring a photon must be 1 and thus the following normalization holds:

$$|\alpha|^2 + |\beta|^2 = 1. \quad (3)$$

The probability $P(\alpha)$ is given by (Born interpretation).

$$P(\alpha) = |\langle \alpha | \Psi \rangle|^2 \quad (4)$$

The modification of a qubit state is described by a matrix. In order to preserve both orthogonality (to ensure easier calculation) and length (preserve total probability) a special matrix is needed, the unitary matrix defined by:

$$A^\dagger \cdot A = I, \quad (5)$$

where $A^\dagger = (A^*)^T$ and I is the identity matrix. Due to Eq. (5) the matrix A must be reversible and thus for a quantum gate (described by a matrix) the number of input pins equals the number of output pins.

4 A few Quantum Gates

A first example of a quantum gate is the CNOT gate, see Fig. 2: qubit q_1 is inverted if qubit $q_0 = |1\rangle$ [4].

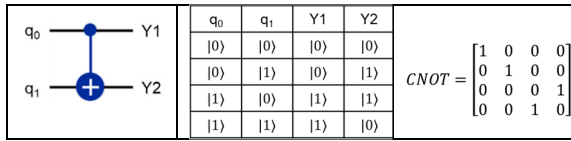


Fig. 2 CNOT = controlled not gate. Left: symbol. Center: truth table. Right: matrix representation.

A second example is the Toffoli or CCNOT gate, shown in Fig. 3: if both $q_0 = q_1 = |1\rangle$, then qubit q_2 is inverted (highlighted in Fig. 3).

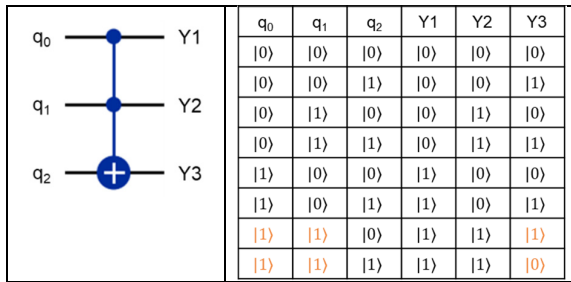


Fig. 3 CCNOT = controlled controlled not gate. Left: symbol. Right: truth table.

In addition, CCNOT enables a XOR operation (sum without carry) if $q_0 = |1\rangle$, then $Y3 = q_1 \oplus q_2$. Also, an AND operation is possible: if $q_2 = |0\rangle$ then $Y3 = q_0 \wedge q_1$.

5 Full Adder

Already Feynman proposed a full adder as quantum computing example [5]. A full adder adds two binary numbers including an input carry and an output carry. Fig. 4 illustrates an example.

Number A		1	0	1	1
Number B		0	1	1	1
Carry	1	1	1	1	-
Sum A+B:		0	0	1	0

Case 4 Case 8

Fig. 4 Adding two binary numbers A and B with carry.

Case	A	B	C_{in}	C_{out}	S
1	$ 0\rangle$	$ 0\rangle$	$ 0\rangle$	$ 0\rangle$	$ 0\rangle$
2	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$
3	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$
4	$ 0\rangle$	$ 1\rangle$	$ 1\rangle$	$ 1\rangle$	$ 0\rangle$
5	$ 1\rangle$	$ 0\rangle$	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$
6	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$	$ 1\rangle$	$ 0\rangle$
7	$ 1\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$
8	$ 1\rangle$	$ 1\rangle$	$ 1\rangle$	$ 1\rangle$	$ 1\rangle$

Tab. 1 Truth table for a full adder (case 4 and 8 are marked).

Starting from most right row: $1+1 = 0$ and carry 1. 2nd row (called Case 8): $1+1+\text{carry } 1 = 1$ and carry 1. The full truth table for all cases of the full adder is given in Tab. 1.

From Tab. 1 the following logic holds for output carry C_{out} and the sum S :

- $C_{out} = [A \wedge B] \vee [C_{in} \wedge (A \oplus B)]$ and
- $S = A \oplus B \oplus C_{in}$.

6 Simulation Results

Feynman's full adder is simulated in Python library Qiskit and the circuit design is shown in Fig. 5 for Case 8. Since Qiskit initializes the qubits always with $|0\rangle$ a Pauli X-gate is needed to convert it into a $|1\rangle$. Thus, after the grey highlighted "boarder" (which has no function – just for better viewing) the Case 8 is generated ($A, B, C_{in} = |1\rangle, |1\rangle, |1\rangle$). By measuring qubit q_2 and q_3 classical bits are generated and results in the output values of Sum and output carry C_{out} .

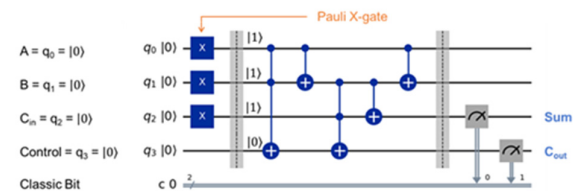


Fig. 5 Quantum circuit for Case 8 compare with Tab. 1.

The simulation result $C_{out}, S = |1\rangle, |1\rangle$ shows Fig. 6.

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In [2]: %runfile C:/User
Results: {'11': 1000}
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Fig. 6 Simulation result for Case 8 and 1000 shots.

7 Summary and Outlook

Based on a few experiments some basics for quantum computing and its math can be understood. As an example, we used the full adder to teach engineers quantum computing and a possible application for it.

References

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- [4] Robert Lored, *Learn Quantum Computing with Python and IBM Quantum Experience*, (Packt Publishing, Birmingham 2020)
- [5] Richard P. Feynman, *Quantum Mechanical Computers in Foundations of Physics*, Vol. 16, No. 6, (1986)