Transimpedance Amplifier as a Second Order Low Pass Filter

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1. The Amplifier

Almost every device for light detection needs a transimpedance converter [1] [2]. Figure 1 shows a simplified version of the electrical representation.

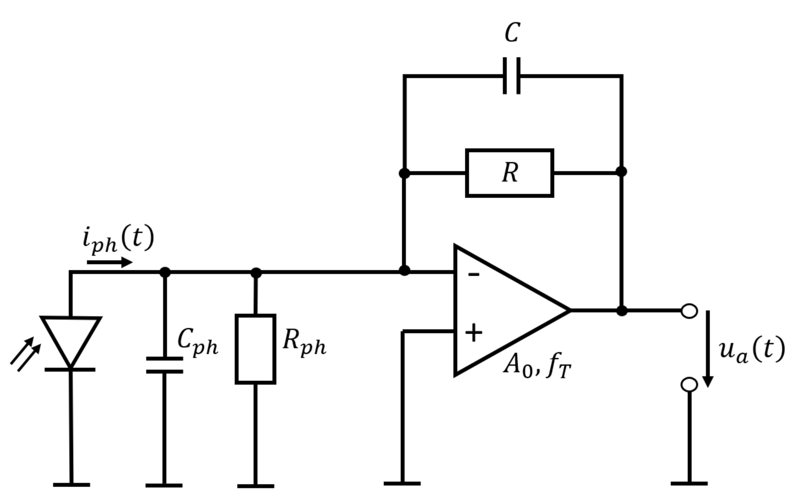

Figure 1: Transimpedance Amplifier

The photocurrent $i_{ph}(t)$ is converted and amplified into a voltage $u_a(t)$ [2] [3]. At this point the frequency response of the system can be specified out of figure 1 [4]:

$$G_{TP2}(s) = \frac{U_a(s)}{I_{ph}(s)} = \frac{K_P \cdot \omega_0^2}{s^2 + 2D\omega_0 \cdot s + \omega_0^2} \quad (1)$$

Equation (1) describes with the help of a very simplified representation the electrical system by means of the Laplace transformation and brings it into a standardized form [2]. It is a second order ($n = 2$) low pass. It allows faster determination of the system's differential equation and represents a transition to the state space:

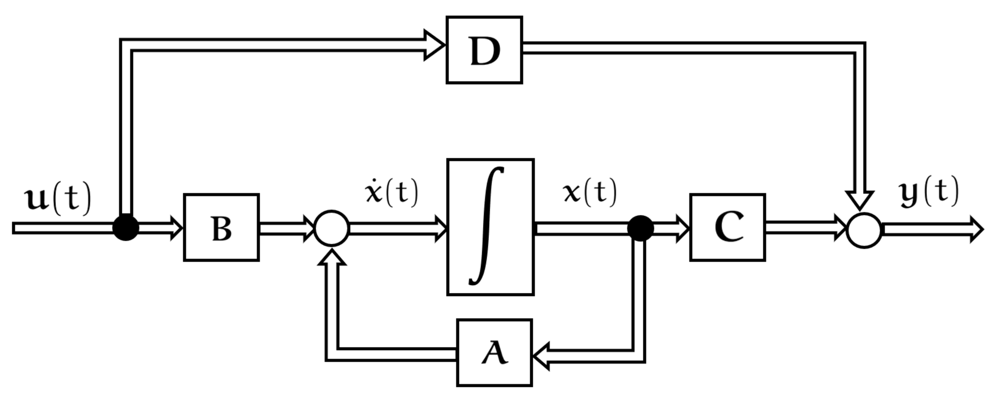

Figure 2: State Space Model

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} \quad (2)$$

$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{u} \quad (3)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ (-(\omega_0)^2) & (-2D\omega_0) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ K_P \cdot \omega_0 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad d = 0$$

2. State Controller

The state space model or description of the system (fig. 2) opens up a whole range of further and, above all, multidimensional control options. The state controller $\mathbf{K}$ feeds back each state under scalar change and in this way it shifts the eigenvalues of the system (fig. 3).

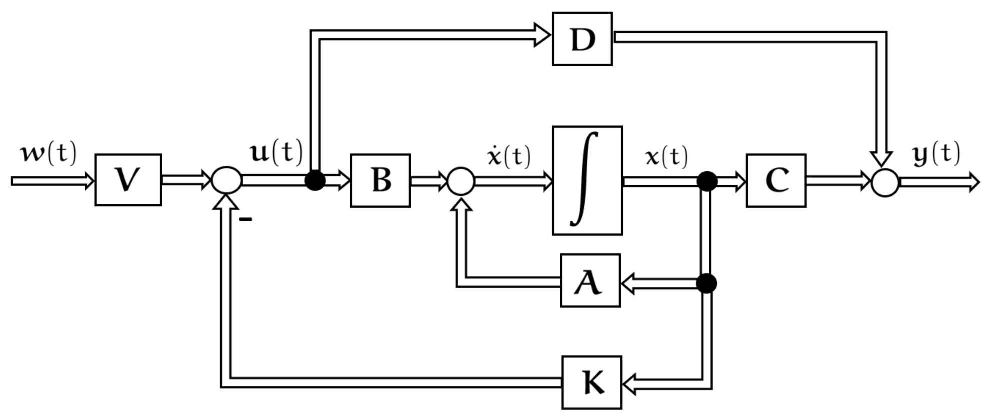

Figure 3: State Space Model and State Controller K

Since the system has only one input signal, it is sufficient to design $\mathbf{K}$ as a single-row control vector $\mathbf{K} \rightarrow \mathbf{k}^T$ and $\mathbf{V}$ becomes a scalar value $\mathbf{V} \rightarrow v$. The feed forward control v of the system is used to scale the output signal as desired [2] [5] [6].

$$\mathbf{t}_{R,1}^T = [0\ 1] \cdot \mathbf{Q}_{Con}^{-1} \quad (4)$$

$$\begin{aligned} P_\alpha(s) &= \Pi_{i=1}^{n}(s - \lambda_i) \\ &= (s - s_{p,1})(s - s_{p,2}) \cdot \ldots \cdot (s - s_{p,n}) \quad (5) \\ &= \alpha_0 + \alpha_1 \cdot s + \ldots + \alpha_{n-1} \cdot s^{n-1} + s^n \end{aligned}$$

$$\mathbf{P}_\alpha(\mathbf{A}) = \alpha_0 \cdot \mathbf{I} + \alpha_1 \cdot \mathbf{A} + \ldots + \alpha_{n-1} \cdot \mathbf{A}^{n-1} + \mathbf{A} \quad (6)$$

$$\mathbf{k}^T = \mathbf{t}_{R,1}^T \cdot \mathbf{P}_\alpha(\mathbf{A}) \quad (7)$$

$$v = \left[\mathbf{c}^T \cdot (\mathbf{b} \cdot \mathbf{k}^T - \mathbf{A})^{-1} \cdot \mathbf{b}\right]^{-1} \cdot \varepsilon \quad (8)$$

By suitable selection of α_i or poles of the system, the dynamics can be directly influenced [2] [5] [6].

3. State Observer

Since not every state can be measured, a Luenberger state observer is used to bridge this information gap [5].

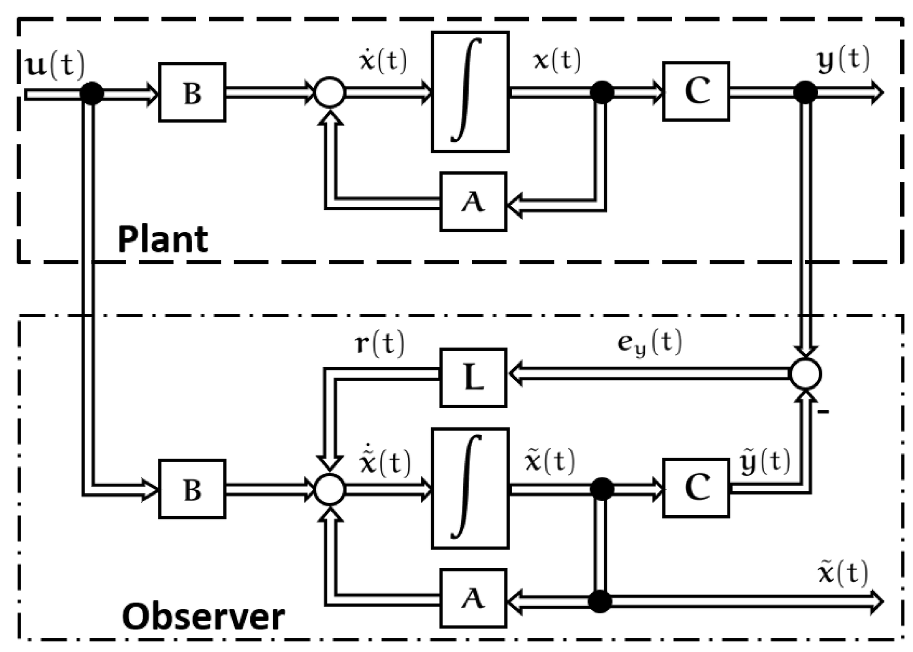

Figure 4: State Space Model & Luenberger Observer

$$\mathbf{t}_{B,1} = \mathbf{Q}_{Obs}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (9)$$

$$\begin{aligned} P_q(s) &= \Pi_{i=1}^{n}(s - \lambda_{B,i}) \\ &= q_0 + q_1 \cdot s + \ldots + q_{n-1} \cdot s^{n-1} + s^n \end{aligned} \quad (10)$$

$$\mathbf{P}_q(\mathbf{A}) = q_0 \cdot \mathbf{I} + q_1 \cdot \mathbf{A} + \ldots + q_{n-1} \cdot \mathbf{A}^{n-1} + \mathbf{A} \quad (11)$$

$$\begin{aligned} \mathbf{l} &= \mathbf{P}_q(\mathbf{A}) \cdot \mathbf{t}_{B,1} \\ &= \left[q_0 \cdot \mathbf{I} + q_1 \cdot \mathbf{A}^1 + \ldots + q_{n-1} \cdot \mathbf{A}^{n-1}\right] \cdot \mathbf{t}_{B,1} \end{aligned} \quad (12)$$

4. Observer & Controller

Using the Luenberger observer, state control can still be performed with high quality by estimating missing states [5].

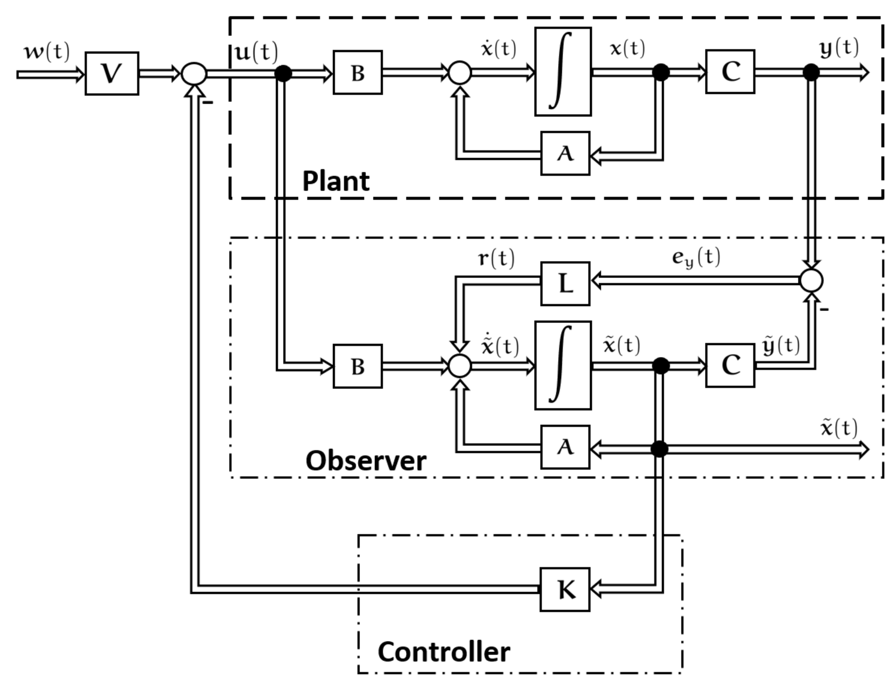

Figure 5: State Observer and Controller

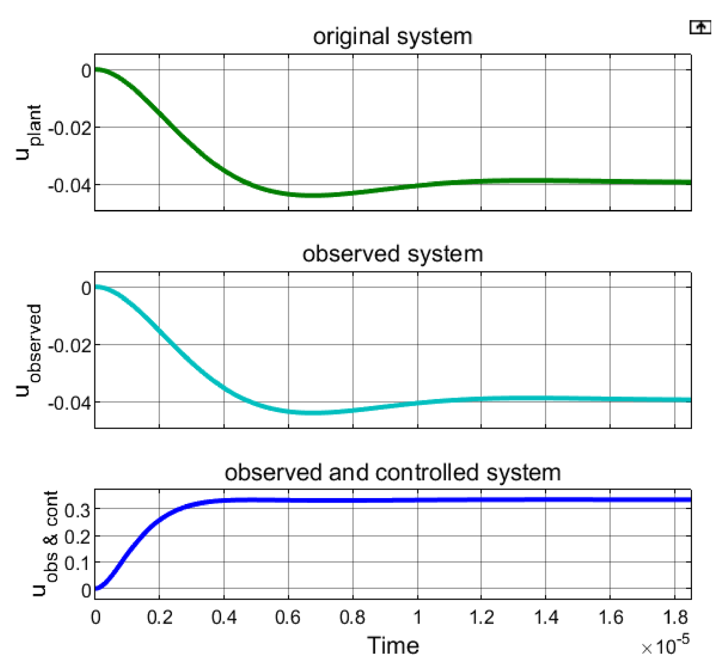

Figure 6: State observed and controlled output voltage signal

The targeted signal improvements result in a cleaner final signal for further processing and increase the quality of the system.

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